Text S2: Dynamic system and equilibria for stages of mosquitoes infected with the fungal pathogen

*(i) Mosquito population dynamics*

*Susceptible mosquitoes*

We now consider susceptible mosquitoes that have been infected with the fungal pathogen for *u* days, in host-seeking stages *i,* . Mosquitoes remain in the first host-seeking stage (*i*=1) for *u* days after contracting the fungus if they do not die due to fungal pathogen-induced mortality or other sources of mortality, and they do not find a blood meal in that time. The probability of this occurring, , is defined in Table S2. The density of mosquitoes in the first host-seeking stage, denoted , is thus given by

 (S2.1)

Host-seeking stages  are denoted , where mosquitoes spend days in stage *i*. These stages are comprised of mosquitoes that entered via three possible routes. Firstly, fungal pathogen-infected mosquitoes that left the *i*-1th host-seeking stage at time when their fungal infection age was , having found a host and not contracted the *Plasmodium* (Table S3). These then survived the *i*-1th non-host-seeking period, lasting days, and remained in the *i*th host-seeking period for time. They have fungal infection age . Secondly, mosquitoes that contracted the fungus during the *i-*1*th* non-host-seeking stage at time *t-u,* survived the rest of this non-host-seeking stage and then survived in the *i*thhost-seeking stage for time  without finding a blood meal. These mosquitoes have a fungal infection age *< u < TM* +. Thirdly, mosquitoes in that contracted the fungus whilst in the *i*th host-seeking stage at time *t-u* and then survived for *u* days without finding a blood meal. For these mosquitoes,  . Thus

 (S2.2)

where  and  are stage-specific probability functions defined in Table S2.

In calculating the density of non-host-seeking mosquitoes that are infected with the fungus we track both the age of the fungal infection, *u,* and the time since the most recent blood meal, *a*, (Table S3). The mosquito density in non-host-seeking stage *i,* denoted , consists firstly of mosquitoes that were in the *i*th host-seeking stage at time *t-a* with fungal infection age *u-a*, at which point they found a blood meal and did not contract the *Plasmodium.* They then survived in the *i*thnon-host seeking stage for *a* days, avoiding fungal-pathogen-induced mortality and other mortality sources, with probability (Table S2). Stages  also consist of mosquitoes that contracted the fungus whilst in the *i*th non-host-seeking stage at time *t-u*. They then survived with the fungal infection for *u* days. Thus,

 (S2.3)

and



(S2.4)

The total densities of susceptible, fungal pathogen-infected mosquitoes in each stage of the gonotrophic cycle are denoted and  for host-seeking and non-host-seeking stages respectively.

 (S2.5)

 (S2.6)

*Exposed mosquitoes*

For exposed, host-seeking mosquitoes that are infected with the fungal pathogen, we track both the *Plasmodium* infection age *p* and the fungal infection age *u.* The density of mosquitoes in the firsthost-seeking stage (*i*=1), denoted , consists of mosquitoes that entered the stage from three possible sources. Firstly, from susceptible, fungal-pathogen infected mosquitoes that found a blood meal and contracted the *Plasmodium* at time *t-p* when their fungal infection age was *u-p* days. These mosquitoes survived the first non-host-seeking stage of the exposed stage, lasting  days, and then avoided mortality due to fungal infection or other mortality sources for *p-* days in the first host-seeking stage, without finding a blood meal. For these mosquitoes *u*>*p.* Secondly, from susceptible mosquitoes uninfected with the fungal pathogen who found a blood meal at time *t-p* and then contracted the fungal pathogen during the first non-host-seeking stage of the exposed stage, at time *t-u.*  They then survived the remainder of this non-host-seeking stage, and survived in the first host-seeking stage for *p-* days without finding a bloodmeal. For these mosquitoes  Lastly, from exposed mosquitoes who contracted the fungal pathogen in the first host-seeking stage at time *p-u*  then survived without finding a blood meal for *u* days. For these mosquitoes  For all mosquitoes in , . Therefore,



(S2.7)

where the stage-specific survival probabilities  and  are defined in Table S2.

For host-seeking stages **, the density of exposed fungal-pathogen infected mosquitoes is denoted  where  is the time spent in the *i*th host-seeking stage (Table S3). Similar to (S2.7), mosquitoes in this stage can also come via three different routes. Firstly, from exposed mosquitoes that found a blood meal in host-seeking stage *i*-1 at time  when their *Plasmodium* infection age was  and their fungal infection age was . They then survived the following non-host-seeking period, lasting days, and then remained in the *i*th feeding cycle for days without finding a blood meal. For these mosquitoes, . Secondly, from exposed mosquitoes uninfected with the fungal pathogen that found a blood meal in host-seeking stage *i*-1 at time  when their *Plasmodium* infection age was days. These mosquitoes then contracted the fungal infection during the *i*th non-host-seeking period at time *t-u*, survived the rest of the non-host-seeking stage, and then survived for days in the *i*th host-seeking stagewithout finding a blood meal. For these mosquitoes, . Lastly, from exposed mosquitoes that became infected with the fungus in the *i*th host-seeking stage, and then survived for a further *u* days without finding a blood meal. For these mosquitoes . For all mosquitoes in , . Therefore,

 (S2.8)

For the first non-host-seeking stage (*i*=1), the density of exposed, fungal-pathogen infected mosquitoes, denoted , consists of mosquitoes from two possible sources. Firstly from susceptible, host-seeking mosquitoes that found a blood meal and contracted the *Plasmodium* at time *t-a* when their fungal infection age was *u-a.* They then survived *a* days in the first non-host-seeking stage. For these mosquitoes,and *u>p.* Secondly, from exposed mosquitoes that caught the fungus in the first non-host-seeking stage of the exposed stage at time *t-u.* They then survived in the first non-host-seeking stage for a further *u* days. For these mosquitoes and *.*



(S2.9)

Densities of fungal pathogen-infected mosquitoes in non-host-seeking stages *i*>1, denoted, are calculated in a similar manner. is composed firstly of mosquitoes that found a blood meal when they were exposed and in host-seeking stage *i*-1 at time *t-a* with *Plasmodium* infection age *p-a* and fungal infection age *u-a.* They then survived *a* days in non-host-seeking stage *i.* For these mosquitoes *u>a* . Secondly, mosquitoes enter by catching the fungus in the *i*th non-host-seeking stage and then surviving in this non-host-seeking stage for a further *u* days. For these mosquitoes *.* For all mosquitoes in  .

 (S2.10)

Total densities of exposed mosquitoes infected with the fungal pathogen are denoted and for host-seeking and non-host-seeking stages respectively.

 (S2.11)

 (S2.12)

*Infectious mosquitoes*

The density of infectious mosquitoes in each host-seeking stage *i* that are infected with the fungal pathogen, denoted , is calculated for each fungal infection age *u* and time spent in the *i*th host seeking stage, . Starting with the first host-seeking stage (*i*=1), mosquitoes in  can come from four possible sources. Firstly, from fungus-infected host-seeking mosquitoes whose *Plasmodium* infection age reached *TE*when their fungal infection age was *u-* at all times *t-*. These mosquitoes then avoided mortality due to fungal infection and other mortality sources for  days without finding a blood meal, with probability  (Table S2). Secondly, from infectious mosquitoes that contracted the fungal pathogen during the first host-seeking stage of the infectious stage, then survived for a further *u* days without finding a blood meal. Thirdly, from non-host-seeking, fungal-pathogen infected mosquitoes whose *Plasmodium* infection age reaches *TE* at days through a non-host-seeking stage. They then survived the remaining days in this non-host-seeking stage, with probability  (Table S2), and go on to survive  days in the first host-seeking stage without finding a blood meal. Lastly, from non-host-seeking mosquitoes uninfected with the fungal pathogen whose *Plasmodium* infection age reaches *TE* at days through a non-host-seeking stage. These mosquitoes then survive the remaining days in this non-host-seeking stage, contracting the fungal pathogen at some point  through this non-host-seeking stage, where . They then survive  days in the first host-seeking stage without finding a blood meal. Thus,

 (S2.13)

Mosquitoes in host-seeking stages *i*>1 come from three possible sources. Firstly, fungal pathogen-infected mosquitoes that found a blood meal in host-seeking stage *i*-1 at time when their fungal infection age was , survived the *i*thnon-host-seeking period then survived for  days in the *i*th non-host-seeking stage without finding a blood meal. Secondly, mosquitoes uninfected with the fungal pathogen that found a blood meal at time and contracted the fungus during the *i*th non-host-seeking stage at time *t-u.* They then survived the rest of the *i*th non-host-seeking stage, and survived for  days in the *i*th non-host-seeking stage without finding a blood meal. Lastly, from mosquitoes that contracted the fungus in the *i*th host-seeking stage at time *t-u*, then survived for *u* days without finding a blood meal.



(S2.14)

To calculate the density of infectious, non-host-seeking mosquitoes that are fungus infected, keep track of the time since the most recent blood meal, *a* (Table S3). The density of mosquitoes in the first non-host-seeking stage (*i*=1), denoted , consists of mosquitoes from two different sources. Firstly, from fungal-pathogen infected, non-host-seeking mosquitoes whose *Plasmodium* infection age reached *TE* at point in a non-host-seeking stage (Table S3). These mosquitoes then survived a further days in the first non-host-seeking stage of the infectious stage, with probability  (Table S2). Secondly, from non-host-seeking mosquitoes without fungal infection whose *Plasmodium* infection age reached *TE* at point in a non-host-seeking stage. These mosquitoes then survived a further days in the first non-host-seeking stage of the infectious stage, contracting the fungal infection at time *t-u*, where . Thus,

 (S2.15)

Mosquito densities in non-host-seeking stages *i>*1, denoted , consist firstly of fungal pathogen-infected mosquitoes that found a blood meal at time *t-a* in the *i-*1th host-seeking stage when their fungal infection age was *u-a*, and then survived *a* days in the *i*th non-host-seeking stage. Secondly, infectious mosquitoes that are without fungal infection and in the *i*thnon-host-seeking stage enter  by contracting the fungal pathogen at time *t-u* and then surviving a further *u* days, *.* Thus,

 (S2.16)

The total densities of malaria infectious mosquitoes infected with the fungal pathogen are denoted and for host-seeking and non-host-seeking stages respectively.

 (S2.17)

 (S2.18)

*(ii) Equilibrium solution*

Expressions for the equilibrium density of susceptible, exposed and infectious host-seeking mosquitoes infected with the fungal pathogen are derived for the limiting case in which  , and . For the case where the shape parameter , fungal infection has a constant linear effect on the mortality rate *μ*, so that . Thus define a constant parameter *c,*  , so that *MF*(*u*) + *μ=c μ*.

*Susceptible mosquitoes*

Beginning with the equilibrium density of susceptible, fungus-infected mosquitoes in the first host-seeking stage, denoted , substitute *MF*(*u*) + *μ=c μ* into equation (S2.1) to give

 (S2.19)

The equilibrium densities in host-seeking stages *i*>1 consists of all mosquitoes that became infected with the fungal pathogen in either host-seeking stages *m*=1,…,*i* or non-host-seeking stages *m*=1,…,*i*-1. These mosquitoes then survived to takeenough blood meals to enter the *ith* host-seeking stage. First, denote the equilibrium density of susceptible, host-seeking mosquitoes in host-seeking stage *i* that caught the fungal infection in host-seeking stage *m* as . These mosquitoessurvived with the fungal infection for ** days, progressing through host-seeking stages *m,…,i* and non-host-seeking stages *m,…,i*-1,taking *i-m* blood meals without contracting the *Plasmodium.* Since the probability of surviving host-seeking stages *m,…,i* is

 (S2.20)

and the probability of surviving non-host-seeking stages *m,…*,*i*-1is , we have

 (S2.21)

Now denote the equilibrium density of susceptible, host-seeking mosquitoes in host-seeking stage *i* that caught the fungal infection in non-host-seeking stage *m* as . For these mosquitoeswe need to consider at which point in the non-host-seeking stage the mosquito contracts the fungus (denoted *a*).



(S2.22)

We now have

 (S2.23)

Similar, but more complex, expressions can be derived for and , the equilibrium densities of fungal pathogen-infected exposed and infectious host-seeking mosquitoes respectively.