Supplementary Text S1

We describe the relation between SDP(2) and SSDP(2).

By identifying SDP(2) with the quotient $\mathrm{GL}(2,\mathbb{R})/\mathrm{O}(2)$, we see that it is also a homogeneous space of the Lie group $\mathrm{GL}(2,\mathbb{R})$ of 2×2 invertible matrices with real coefficients. It is useful to consider the symmetric space of special symmetric positive matrices $\mathrm{SSDP}(2) = \mathrm{SDP}(2) \cap \mathrm{SL}(2,\mathbb{R}) = \{A \in \mathrm{SDP}(2), \det A = 1\}$. This submanifold can also be identified with the quotient $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$, which is itself isomorphic to the hyperbolic space H_2. Here $\mathrm{SL}(2,\mathbb{R})$ denotes the special linear group of all determinant one matrices in $\mathrm{GL}(2,\mathbb{R})$. Therefore SSDP(2) is a totally geodesic submanifold of SDP(2) [1]. Now since $\mathrm{SDP}(2) = \mathrm{SSDP}(2) \times \mathbb{R}^+$, it can be seen as a foliated manifold whose codimension-one leaves are isomorphic to the hyperbolic surface H^2.

References

1. Maass H (1971) Siegel's Modular Forms and Dirichlet Series. Lecture Notes in Mathematics 216. Springer-Verlag, Heidelberg.