## Supplementary Text S2

The Poincaré half-plane model, noted $\mathcal{H}$, is obtained from the Poincaré disk model by the mapping $f$ such that

$$
u=f(z)=-i \frac{z+1}{z-1}
$$

which is an isometry from $D$ to the upper half-plane $\mathcal{H}:\{\operatorname{Im}(z)>0\}$. The distance between two points $u, u^{\prime}$ in $\mathcal{H}$ is then easily obtained from the distance in $D$ by setting $z=f^{-1}(u)$ and $z^{\prime}=f^{-1}\left(u^{\prime}\right)$ in the expression (7) in the main text. This gives

$$
d_{3}\left(u, u^{\prime}\right)=d_{2}\left(f^{-1}(u), f^{-1}\left(u^{\prime}\right)\right)=\operatorname{arctanh} \frac{\left|u^{\prime}-u\right|}{\left|u^{\prime}-\bar{u}\right|}
$$

Geodesics in $\mathcal{H}$ are lines or circles orthogonal to the real axis. The surface element in $H^{2}$ is

$$
d s^{2}=u_{2}^{-2}\left(d u_{1}^{2}+d u_{2}^{2}\right)
$$

if $u=u_{1}+i u_{2}$.

