## Supplementary Text S2

The Poincaré half-plane model, noted  $\mathcal{H}$ , is obtained from the Poincaré disk model by the mapping f such that

$$u = f(z) = -i\frac{z+1}{z-1}$$

which is an isometry from D to the upper half-plane  $\mathcal{H}$ : {Im(z) > 0}. The distance between two points u, u' in  $\mathcal{H}$  is then easily obtained from the distance in D by setting  $z = f^{-1}(u)$  and  $z' = f^{-1}(u')$  in the expression (7) in the main text. This gives

$$d_3(u, u') = d_2(f^{-1}(u), f^{-1}(u')) = \operatorname{arctanh} \frac{|u' - u|}{|u' - \overline{u}|}$$

Geodesics in  $\mathcal{H}$  are lines or circles orthogonal to the real axis. The surface element in  $H^2$  is

$$ds^2 = u_2^{-2}(du_1^2 + du_2^2),$$

if  $u = u_1 + iu_2$ .