

# Do Humans Optimally Exploit Redundancy to Control Step Variability in Walking?

Jonathan B. Dingwell, Joby John and Joseph P. Cusumano

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## SUPPLEMENTARY TEXT #S1

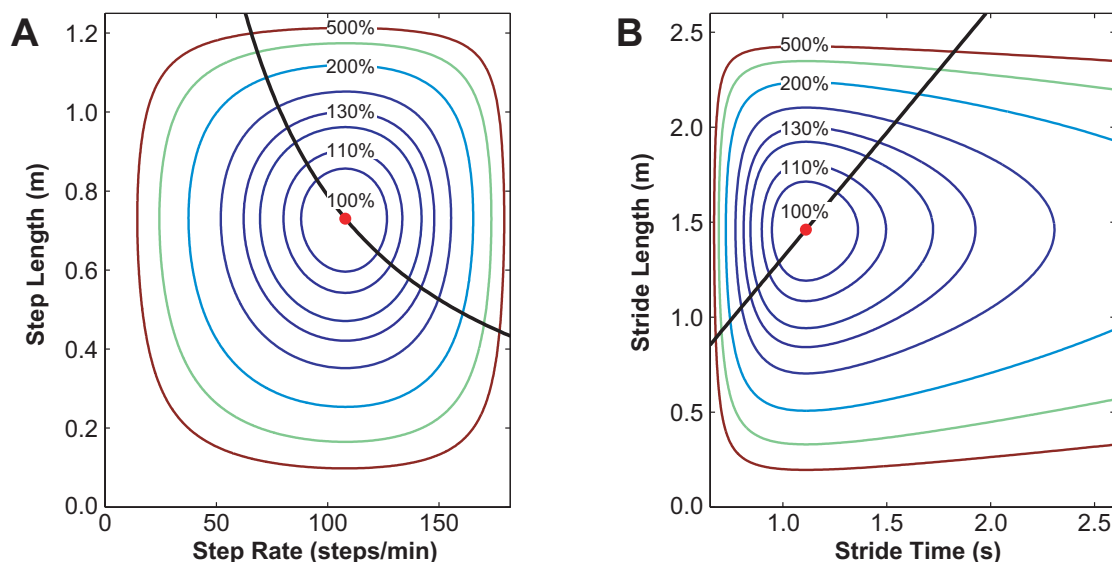
### Extended Description of the Construction of Figure 1

Zarrugh et al. [85] used experimental data collected during treadmill walking across a wide range of imposed speeds, stride lengths, and stride frequencies to derive detailed expressions for predicting metabolic energy cost as a function of *step* length ( $s$ ) and *step* frequency ( $f$ ). The expression they derived for the energy expenditure per meter walked per kg body mass (combining their Eq. 2 and Eq. 4 from [85]) is:

$$E_m(s, f) = \frac{E_w(s, f)}{v} = \frac{E_0}{s f (1 - s^2/s_u^2)(1 - f^2/f_u^2)} = \frac{s_u^2 f_u^2 E_0}{s f (s_u^2 - s^2)(f_u^2 - f^2)}, \quad (7)$$

where  $s$  is average step length (i.e.,  $s = L/2$  in meters),  $f$  is average step *rate* in steps/minute (i.e.,  $f = (1/T) \times 60$ ), and  $E_0$ ,  $s_u$  and  $f_u$  are all constants that must be derived experimentally for individual subjects [85].

To demonstrate how these contour curves behave, we replicate the results of Zarrugh for one typical subject for whom they obtained experimental values of  $E_0$ ,  $s_u$  and  $n_u$ . Fig. 10A replicates their Fig. 6b [85], showing energy cost ( $E_m$ ) as a function of *average step rate* and *average step length*. Fig. 10B shows the same energy contours, plotted in terms of *average stride time* and *average stride length* (i.e.,  $[T, L]$ ) as in the manner of our paper (Fig. 1). In both plots, black diagonal lines indicate the energetically optimal walking speed (in this case, 1.314 m/s) that would indicate the GEM. As shown in Fig. 1 in the main manuscript, near the optimum value, comparable small relative changes in either stride length or stride time result in comparable changes in energy cost.



**Figure 10 – Energy cost ( $E_m$ ) contours for a typical subject.** Panel (A) is plotted in the same manner as Zarrugh et al., 1974 (Ref. [85], their Fig. 6b). Panel (B) shows the same data plotted in the manner of Fig. 1 from this manuscript.

## References:

85. Zarrugh MY, Todd FN, Ralston HJ (1974) Optimization of Energy Expenditure During Level Walking. *European Journal of Applied Physiology* 33: 293-306.