

Do Humans Optimally Exploit Redundancy to Control Step Variability in Walking?

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SUPPLEMENTARY TEXT #S4

Extended Description of the Detrended Fluctuation Analysis Algorithm

Detrended Fluctuation Analysis (DFA; Fig. 13) [99-102] was used to determine the degree to which each time series exhibited persistent or anti-persistent temporal correlations across successive strides. DFA has been used extensively to analyze physiological time series because it is robust to noise and nonstationarities [100]. Complete details of the DFA algorithm are published elsewhere [99-102].

In brief, each time series $x(n)$, where $n \in \{1, \dots, N\}$ strides, was first integrated to form a cumulative sum:

$$y(k) = \sum_{i=1}^k [x(i) - \bar{x}] \quad (24)$$

where $x(n)$ was the value of x for the n^{th} stride and \bar{x} was the mean value of x across all N strides. Each integrated series was divided into equal, non-overlapping segments of length j . Each segment was detrended by subtracting a least squares linear fit to that segment (Fig. 13B). The squares of the residuals were then averaged over the entire data set and the square root of the mean residual, $F(j)$, was calculated:

$$F(j) = \sqrt{\frac{1}{N} \sum_{k=1}^N [y(k) - y_{fit}(k)]^2} \quad (25)$$

This process was repeated for different segment lengths, j . Fifty values of j that were evenly distributed between 4 and $N/4$ were used. Typically, $F(j)$ increases with j and a graph of $\log[F(j)]$ versus $\log(j)$ will exhibit a power-law relationship indicating the presence of scaling (Fig. 13C), such that $F(j) \approx j^\alpha$ [99-101]. These $\log[F(j)]$ versus $\log(j)$ plots were then fit with a linear function using least squares regression. The slope of this line defined the scaling exponent α (Fig. 13C). A value of $\alpha = 0.5$ indicates that the time series is uncorrelated white noise. When $\alpha < 0.5$, the time series contains anti-persistent temporal correlations. When $\alpha > 0.5$, persistent temporal correlations exist [99,100]. When $\alpha = 1.5$, the time series is brown noise (i.e., integrated white noise) [99,100]. Although one could alternatively perform higher order (e.g., quadratic) detrending, this did not alter results obtained from similar time series [103,104].

References:

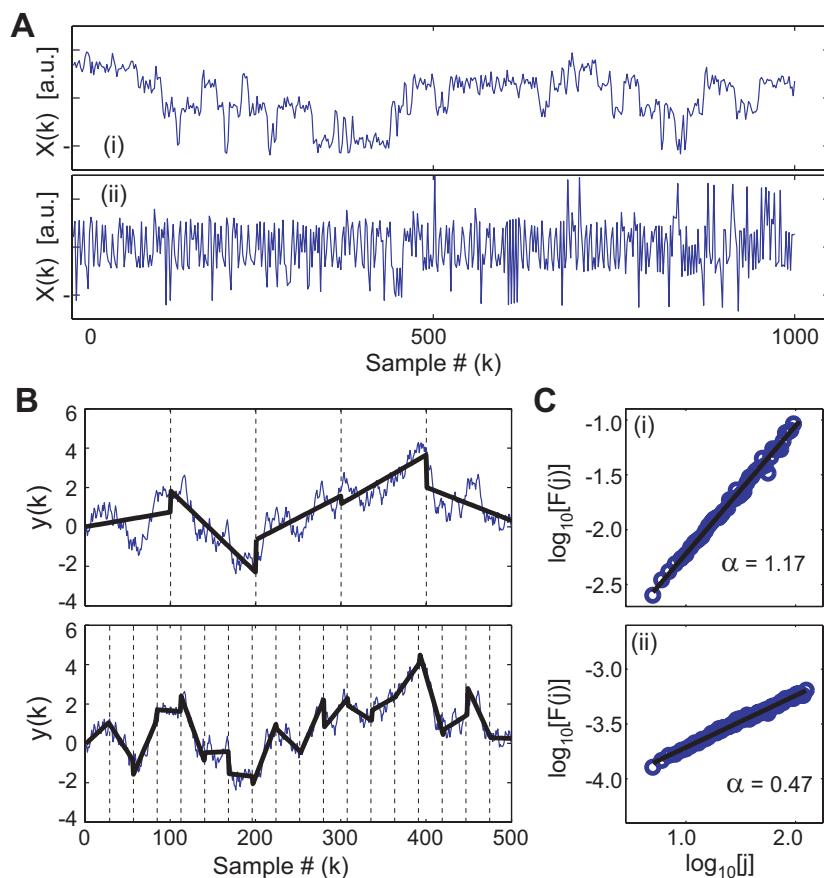
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Figure 13 – Illustration of detrended fluctuation analysis.

(A) Examples of two simulated stochastic time series, $x(k)$, in arbitrary units (a.u.) with equal variance but very different temporal correlation structures. The top graph (i) clearly shows temporal persistence, whereas the bottom graph (ii) fluctuates much more rapidly.

(B) Each time series was first integrated (thin blue lines, Eq. S5-1) and then divided into bins, where each bin contains j samples. Data within each bin were fitted with a straight line (thick black lines) using linear regression. This linear fit was then subtracted from the original and the root mean square of the residuals was calculated (Eq. S5-2). This process was repeated for a range of bins lengths between 4 and 125. As bin size decreased the error also decreased.

(C) DFA analyses of the two corresponding time series shown in A. The different temporal correlation patterns are clearly reflected in the different scaling exponent values obtained ($\alpha = 1.17$ vs. $\alpha = 0.47$). Thus, the α exponent obtained from DFA quantifies temporal correlations (persistence vs. anti-persistence) in a time series, independent of the magnitude of the variance.



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