

## Text S1: Derivation of the steady state firing rate $R^*$

In the steady state (Phase III) we can assume that the calcium concentration of each neuron  $c_i(t)$  is on average (measured over longer times) equal to  $c_{target}$ . Furthermore, we assume a flat distribution for the firing, hence averaging across avalanches and pauses. This way we can define  $c_i(t) \forall t \in (t_i^{sp}, t_i^{sp} + t_{max})$  via:

$$c_i(t) = c_i(t_i^{sp}) \cdot e^{-\frac{t-t_i^{sp}}{\tau_C}}, \quad (1)$$

where  $t_i^{sp}$  are the moments when a neuron fires and  $t_{max}$  the interval between two subsequent spikes. We get:

$$c_i(t_i^{sp}) = c_i(t_{max}) + \beta = c_i(t_i^{sp}) \cdot e^{-\frac{t_{max}}{\tau_C}} + \beta \quad (2)$$

$$\rightarrow c_i(t_i^{sp}) = \frac{\beta}{1 - e^{-\frac{t_{max}}{\tau_C}}} \quad (3)$$

$$\Rightarrow c_i(t) = \frac{\beta}{1 - e^{-\frac{t_{max}}{\tau_C}}} \cdot e^{-\frac{t-t_i^{sp}}{\tau_C}} \quad (4)$$

$$\langle c_i(t) \rangle = \frac{1}{t_{max}} \int_{t_i^{sp}}^{t_i^{sp}+t_{max}} c_i(t) dt \quad (5)$$

$$= \frac{\beta}{t_{max}(1 - e^{-\frac{t_{max}}{\tau_C}})} \int_{t_i^{sp}}^{t_i^{sp}+t_{max}} e^{-\frac{t-t_i^{sp}}{\tau_C}} dt \quad (6)$$

$$= \frac{\beta \cdot \tau_C}{t_{max}(1 - e^{-\frac{t_{max}}{\tau_C}})} \left(1 - e^{-\frac{t_{max}}{\tau_C}}\right) \quad (7)$$

$$= \frac{\beta \cdot \tau_C}{t_{max}} \stackrel{!}{=} c_{target} \quad (8)$$

As average rate  $R^*$  is inverse to the average firing interval we finally arrive at:

$$R^* = \frac{c_{target}}{\beta \cdot \tau_C}. \quad (9)$$