

Supplemental Material

Computation of Unexpected Uncertainty

To compute $\lambda(T)_{\text{red}} = P(J_{\text{red}T} = 0 | \underline{\mathbf{c}}_{\text{IT}})$, let $P_{lT}(\mathbf{p})(J_{\text{red}T} = 0)$ and $P_{lT}(\mathbf{p})(J_{\text{red}T} = 1)$ denote the posterior probability distribution after *no jump* and after *jump*, respectively. Absent further information, the Bayesian model sets the prior belief of a jump equal to 0.5. Thus, the subjective jump likelihood at time T , $P(J_{\text{red}T} = 0 | \underline{\mathbf{c}}_{\text{IT}})$, equals to

$$\frac{1/2 \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT}(\mathbf{p})(J_{\text{red}T} = 0) d\mathbf{p}}{1/2 \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT}(\mathbf{p})(J_{\text{red}T} = 0) d\mathbf{p} + 1/2 \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT}(\mathbf{p})(J_{\text{red}T} = 1) d\mathbf{p}}.$$

We can rewrite the previous form as

$$\frac{1}{1 + \frac{\int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT}(\mathbf{p}) (J_{\text{red}T} = 1) d\mathbf{p}}{\int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT}(\mathbf{p}) (J_{\text{red}T} = 0) d\mathbf{p}}}},$$

where

$$\begin{aligned} P_{lT}(\mathbf{p})(J_{\text{red}T} = 1) &= P_0(\mathbf{p}), \\ P_{lT}(\mathbf{p})(J_{\text{red}T} = 0) &= P_{lT/T}(\mathbf{p}) \left(= \frac{l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT-1}(\mathbf{p})}{\int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT-1}(\mathbf{p}) d\mathbf{p}}\right). \end{aligned}$$

Therefore, we have:

$$\lambda_{\text{red}}(T) = \frac{1}{1 + \frac{AC}{B}},$$

where

$$\left\{ \begin{array}{l} A = \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_0(\mathbf{p}) d\mathbf{p}, \\ B = \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p})^2 P_{lT-1}(\mathbf{p}) d\mathbf{p}, \\ C = \int_{\Theta} l(\mathbf{c}_{\text{IT}} | \mathbf{p}) P_{lT-1}(\mathbf{p}) d\mathbf{p}. \end{array} \right.$$

Since $l(\mathbf{c}_{lT} | \mathbf{p}) = \prod_{i=1}^3 p_i^{c_{liT}}$, we write A as follows:

$$A = \int_{\Theta} \prod_{i=1}^3 p_i^{c_{liT}} \frac{\prod_{i=1}^3 p_{liT}^{\nu_0 \hat{p}_{i0}-1}}{\prod_{i=1}^3 \Gamma(\nu_0 \hat{p}_{i0})} d\mathbf{p}.$$

Because

$$\int_{\Theta} \prod_{i=1}^3 p_i^{c_{liT} + \nu_0 \hat{p}_{i0}-1} d\mathbf{p} = \frac{\prod_{i=1}^3 \Gamma(c_{liT} + \nu_0 \hat{p}_{i0})}{\Gamma(\sum_{i=1}^3 c_{liT} + \nu_0)},$$

we can rewrite A as follows:

$$A = \frac{\Gamma(\nu_0) \prod_{i=1}^3 \Gamma(c_{liT} + \nu_0 \hat{p}_{i0})}{\underbrace{\Gamma(\sum_{i=1}^3 c_{liT} + \nu_0)}_1 \prod_{i=1}^3 \Gamma(\nu_0 \hat{p}_{i0})}.$$

Let i^* refer to the realized component of the count vector at time $T - 1$. (For example, suppose that location l delivered the loss outcome at trial $T - 1$; then $c_{lT-1} = (1, 0, 0)$, and i^* is equal to 1.) Because $\Gamma(x + 1) = x\Gamma(x)$, and since $c_{liT} = 0$, $\forall i \neq i^*$ while $c_{li^*T} = 1$, we can further simplify this expression:

$$A = \frac{\Gamma(\nu_0) \prod_{i=1}^3 \Gamma(c_{liT} + \nu_0 \hat{p}_{i0})}{\Gamma(1 + \nu_0) \prod_{i=1}^3 \Gamma(\nu_0 \hat{p}_{i0})} = \frac{\prod_{i=1}^3 \Gamma(c_{liT} + \nu_0 \hat{p}_{i0})}{\nu_0 \prod_{i=1}^3 \Gamma(\nu_0 \hat{p}_{i0})}.$$

Hence,

$$\begin{aligned} A &= \frac{\Gamma(1 + \nu_0 \hat{p}_{i^*0}) \prod_{i \neq i^*} \Gamma(\nu_0 \hat{p}_{i0}) \Gamma(1 + \nu_0 \hat{p}_{i^*0})}{\nu_0 \prod_{i=1}^3 \Gamma(\nu_0 \hat{p}_{i0})} \\ &= \frac{1}{\nu_0} \frac{\Gamma(1 + \nu_0 \hat{p}_{i^*0})}{\Gamma(\nu_0 \hat{p}_{i^*0})} \\ &= \hat{p}_{i^*0}. \end{aligned}$$

The calculation of B is analogous:

$$B = \frac{\Gamma(\nu_{lT-1}) \prod_{i=1}^3 \Gamma(2c_{liT} + \nu_{lT-1}\hat{p}_{ilT-1})}{\Gamma(2 \underbrace{\sum_{i=1}^3 c_{liT} + \nu_{lT-1}}_1) \prod_{i=1}^3 \Gamma(\nu_{lT-1}\hat{p}_{ilT-1})}.$$

Since

$$\Gamma(2 + \nu_{lT-1}) = \Gamma(1 + (1 + \nu_{lT-1})) = (1 + \nu_{lT-1})\Gamma(1 + \nu_{lT-1}) = (1 + \nu_{lT-1})\nu_{lT-1}\Gamma(\nu_{lT-1}),$$

and

$$\prod_{i=1}^3 \Gamma(2c_{liT} + \nu_{lT-1}\hat{p}_{ilT-1}) = \prod_{i \neq i^*} \Gamma(\nu_{lT-1}\hat{p}_{ilT-1})(1 + \nu_{lT-1}\hat{p}_{i^*lT-1})\Gamma(1 + \nu_{lT-1}\hat{p}_{i^*lT-1}),$$

B simplifies to:

$$B = \frac{(1 + \nu_{lT-1}\hat{p}_{i^*lT-1})\hat{p}_{i^*lT-1}}{1 + \nu_{lT-1}}.$$

Using analogous arguments,

$$C = \frac{\Gamma(\nu_{lT-1}) \prod_{i=1}^3 \Gamma(c_{liT} + \nu_{lT-1}\hat{p}_{ilT-1})}{\Gamma(2 \underbrace{\sum_{i=1}^3 c_{liT} + \nu_{lT-1}}_1) \prod_{i=1}^3 \Gamma(\nu_{lT-1}\hat{p}_{ilT-1})} = \hat{p}_{i^*lT-1}.$$

Consequently,

$$\frac{AC}{B} = \frac{\hat{p}_{i^*0}(\nu_{lT-1} + 1)}{1 + \nu_{lT-1}\hat{p}_{i^*lT-1}}.$$

Thus, $\lambda(T)$ depends on $\frac{\hat{p}_{i^*0}}{\hat{p}_{i^*lT-1}}$, the *odds ratio* (i.e., *strength of evidence*) for the hypothesis that a jump has occurred at time T .