# Benchmarking Ontology Supplement 

October 21, 2010

## Contents

1 Information retrieval metrics ..... 1
2 Novel metrics for evaluating ontology fitness ..... 2
3 The fittest ontology of given size ..... 4
4 Comparing corpora ..... 5
5 Data ..... 7
6 Results ..... 9

## 1 Information retrieval metrics

In the field of information retrieval (IR), the goal is to identify documents from a large collection that are most relevant to a user's query. If the subset of relevant documents is known, we can evaluate the quality of an information retrieval method using the metrics of precision, recall, accuracy, fallout and the F-measure (harmonic mean of precision and recall). To define these metrics we need to determine the true positives ( $t p$ ), false positives $(f p)$, true negatives $(t n)$, and false negatives ( $f n$ ) achieved by the retrieval method. These are defined by the cross-tabulation between relevance and retrieval: true positives comprise documents that are both relevant to the query and retrieved by the method; false positives are documents retrieved but irrelevant; false negatives are relevant but not retrieved; and true negatives include all irrelevant documents not retrieved by the method.

In the case of synonym thesauri, all the synonym pairs happening in the processed thesauri can be grouped into four categories similarly: true positives which refer to synonym pairs that occur in both a given thesaurus and a given corpus, false positives which occur in the thesaurus but not the corpus, false negatives which occur in the corpus but not the thesaurus (but perhaps in some other thesaurus), and true negatives which occur in neither the thesaurus nor the corpus. As discussed in the main text, such a simple transfer
of definition to ontology has issues. However we computed the following IR metrics based on these definitions, mainly for a comparison with our proposed ontology-evaluation metrics.

Based on the above definitions, some common metrics used in IR are defined as follows:

$$
\begin{gather*}
\text { Precision } \stackrel{\text { def }}{=} \frac{N_{t p}}{N_{t p}+N_{f p}},  \tag{1}\\
\text { Recall } \stackrel{\text { def }}{=} \frac{N_{t p}}{N_{t p}+N_{f n}},  \tag{2}\\
\text { Accuracy } \stackrel{\text { def }}{=} \frac{N_{t p}+N_{t n}}{N_{t p}+N_{t n}+N_{f p}+N_{f n}},  \tag{3}\\
\text { Fallout } \stackrel{\text { def }}{=} \frac{N_{f p}}{N_{t n}},  \tag{4}\\
F \stackrel{\text { def }}{=} 2 \cdot \frac{\text { Precision } \cdot \text { Recall }}{\text { Precision }+ \text { Recall }},  \tag{5}\\
F_{\beta} \stackrel{\text { def }}{=}\left(1+\beta^{2}\right) \cdot \frac{\text { Precision } \cdot \text { Recall }}{\left(\beta^{2} \cdot \text { Precision }+ \text { Recall }\right)} \tag{6}
\end{gather*}
$$

$F_{\beta}=F$ when $\beta=1 . F_{2}$ weights recall twice as much as precision and $F_{\frac{1}{2}}$ weights precision twice as much as recall. The results are listed in Table 2 in the Results section.

## 2 Novel metrics for evaluating ontology fitness

For a given reference corpus $T$, we define the complete ontology $\mathcal{O}\left(\mathcal{C}_{T}, \mathcal{R}_{T}\right)$ which incorporates all $N$ concepts encountered in the corpus and all the relations between them. We also derive from the corpus $T$, a frequency $f_{i}$ for each concept in $\mathcal{C}_{T}$ and an association probability $p_{i j}$ for each relation in $\mathcal{R}_{T} . f_{i}$ should be normalized in such a way that $\sum_{i \in C_{T}} f_{i}=\sum_{i=1}^{N} f_{i}=1$ and by definition (See section one), $p_{i j}$ is normalized so that $\sum_{j=1}^{M_{i}} p_{i j}=1$ for a given concept $i$. In implementation, we (under)approximate the complete
ontology (thesaurus) with the union of thesauri, excluding concepts and relations not found in the corpus.

To evaluate an arbitrary ontology, $X=\left\{C_{X}, R_{X}\right\}$, regarding to corpus $T$, we can identify sets $C_{X}(t p)$, and $R_{X}(t p)$, such that $C_{X}(t p)=C_{X} \cap \mathcal{C}_{T}$, and $R_{X}(t p)=R_{X} \cap \mathcal{R}_{T}$.

This allows us to replace integer $N_{t p}$ with real-valued weight $W_{C_{X}(t p)}$ such that

$$
\begin{equation*}
W_{C_{X}(t p)}(T) \stackrel{\text { def }}{=} \sum_{i \in C_{X}(t p)} f_{i}, \tag{7}
\end{equation*}
$$

If we expand this measure to also account for relation importance, it becomes

$$
\begin{equation*}
W_{C_{X}(t p) R_{X}(t p)}(T) \stackrel{\text { def }}{=} \sum_{i \in C_{X}(t p)} \sum_{j \in C_{X}(t p)} \sum_{k(i) \in R_{X}(t p)} f_{i} p_{k \mid i j}, \tag{8}
\end{equation*}
$$

where $p_{k \mid i j}$ is equal to the association probability between concepts $i$ and $j, p_{i j}$, if a relation between them exists in $X$, and is zero otherwise.

Similarly we define $C_{X}(f n)=C_{T}-C_{X}(t p)$ and $R_{X}(f n)=R_{T}-R_{X}(t p)$ and get

$$
\begin{equation*}
W_{C_{X}(f n) R_{X}(f n)}(T) \stackrel{\text { def }}{=} \sum_{i \in C_{X}(f n)} \sum_{j \in C_{X}(f n)} \sum_{k \in R_{X}(f n)} f_{i} p_{k \mid i j}, \tag{9}
\end{equation*}
$$

Now we are able to introduce our first ontology-evaluation measure -breadth- to capture the theoretical coverage of an ontology:
$\operatorname{Breadth}_{X}^{2}(T) \stackrel{\text { def }}{=} \frac{\sum_{i \in C_{X}(t p)} \sum_{j \in C_{X}(t p)} \sum_{k \in R_{X}(t p)} f_{i} p_{k \mid i j}}{\sum_{i^{\prime} \in C_{X}(t p)} \sum_{j^{\prime} \in C_{X}(t p)} \sum_{k^{\prime} \in R_{X}(t p)} f_{i^{\prime}} p_{k^{\prime} \mid i^{\prime} j^{\prime}}+\sum_{i^{\prime \prime} \in C_{X}(f n)} \sum_{j^{\prime \prime} \in C_{X}(f n)} \sum_{k^{\prime \prime} \in R_{X}(f n)} f_{i^{\prime \prime}} p_{k^{\prime \prime} \mid i^{\prime \prime} j^{\prime \prime}}}$

Because every concept and its relations in a corpus either happen in the ontology $(t p)$ or not ( $f n$ ), equation (10) can be simplified as follows:

$$
\begin{align*}
& \text { Breadth }_{X}{ }^{2}(T)=\frac{\sum_{i \in C_{X}(t p)}}{\sum_{i^{\prime} \in C_{T}} \sum_{j \in C_{X}(t p)} \sum_{j^{\prime} \in C_{T}} \sum_{k^{\prime} \in R_{T}} f_{i^{\prime}(t p)} f_{i} p_{k \mid i j}} p_{k^{\prime} \mid i^{\prime} j^{\prime}} \\
&=\frac{\sum_{i \in C_{X}(t p)} \sum_{j \in C_{X}(t p)} \sum_{k \in R_{X}(t p)} f_{i} p_{k \mid i j}}{\sum_{i^{\prime} \in C_{T}} f_{i^{\prime}} \sum_{j^{\prime} \in C_{T}} p_{k^{\prime} \mid i^{\prime} j^{\prime}}} \\
&=\sum_{k^{\prime} \in R_{T}} \sum_{i \in C_{X}(t p)} \sum_{j \in C_{X}(t p)} f_{i} p_{k \mid i j}  \tag{11}\\
&=W_{C_{X}(t p) \in R_{X}(t p)}  \tag{12}\\
& R_{X}(t p)
\end{align*}
$$

This approach of weighing importance works as intended for $N_{t p}$ and $N_{f n}$, but not for $N_{f p}$ and $N_{t n}$ because the corresponding $f_{i}$ 's all equal zero in the corpus.

We can further modify this measure of theoretical coverage to account also for parsimony, and thus develop a general measure of Depth of ontology $X$ with respect to corpus $T$ :

$$
\begin{align*}
\operatorname{Depth}_{X}^{2}(T) & \stackrel{\text { def }}{=} \frac{\operatorname{Breadth}_{X}^{2}(T)}{\text { Number of relations in } X}  \tag{13}\\
& =\frac{\sum_{i \in C_{X}(t p)} \sum_{j \in C_{X}(t p)} \sum_{k \in R_{X}(t p)} f_{i} p_{k \mid i j}}{\left|R_{X}\right|} . \tag{14}
\end{align*}
$$

In the case of an ontology, Depth translates into the average probability mass (in a corpus) for each concept relation. Large ontologies would tend to have a better value of Breadth, but not necessarily a better Depth. This is because a large ontology may be padded with very rare concepts and relations lowering its fit to the corpus compared to a small ontology containing only the most frequent ones.

Finally, we can create a more general measure $\operatorname{Depth}_{\beta}$ that allows flexibility in the specification of ontological coverage and parsimony, such that

$$
\begin{equation*}
\operatorname{Depth}_{X, \beta}(T)=\frac{\left[\operatorname{Breadth}_{X}\right]^{(2-\beta)}}{\left|R_{X}\right|^{\beta}} \tag{15}
\end{equation*}
$$

In implementation, we tried $\beta=0.5,0.75,1.5$ for this equation. The results are presented in Table 2 of the Results section.

## 3 The fittest ontology of given size

We can then define the fittest ontology of fixed size, $\mathcal{O}_{c r}\left(T, C, R,\left\{f_{i},\left\{p_{i j}\right\}_{j=1, \ldots, M_{i}^{\prime}}\right\}_{i=1, \ldots, c}\right)$ with a predetermined $c$ concepts and $r$ relations $\left(r=\sum_{i \in c} M_{i}^{\prime}\right)$ such that $C \subset \mathcal{C}_{\mathcal{T}}, R \subset \mathcal{R}_{\mathcal{T}}$, and Breadth $h_{\mathcal{O}_{c r}}(T)$ is maximized over all possible sets $C$ and $R$ of sizes $c$ and $r$, correspondingly.

For an arbitrary ontology $O_{c r}$, we would like to benchmark it using the fittest ontology of the same size, $\mathcal{O}_{c r}$. Once we have estimated its Breadth ${O_{c r}}$ and Depth ${ }_{O_{c r} r}$ for a given corpus $T$, we can compute the loss measures relative to its fittest counterpart:

$$
\begin{gather*}
\operatorname{Breadth}_{\operatorname{Loss}}^{O_{c r}}  \tag{16}\\
(T)=\operatorname{Breadth}_{\mathcal{O}_{c r}}(T)-\operatorname{Breadth}_{O_{c r}}(T),  \tag{17}\\
{\operatorname{Depth} \operatorname{Loss}_{O_{c r}}(T)=\operatorname{Depth}_{\mathcal{O}_{c r}}(T)-\operatorname{Depth}_{O_{c r}}(T) .}^{\text {(T) }} .
\end{gather*}
$$

To ease computation, we can define simplified versions of these measures that constrain only the number of relations, $r$ :

$$
\begin{gather*}
{\operatorname{Breadth} \operatorname{Loss}_{O_{* r}}(T)=\operatorname{Breadth}_{\mathcal{O}_{* r}}(T)-\operatorname{Breadth}_{O_{* r}}(T),}_{\operatorname{Depth}_{\operatorname{Loss}_{O_{* r}}}(T)=\operatorname{Depth}_{\mathcal{O}_{* r}}(T)-\operatorname{Depth}_{O_{* r}}(T),}, \tag{18}
\end{gather*}
$$

where $*$ indicates that $c$ is not constrained. These results are also summarized in Table 2.
The strength of the loss measure is its ability to compare a specific ontology to the Depth-optimized ontology of the same size, rather than one significantly larger or smaller. In theory, this could allow us to benchmark ontologies covering domains for which there may be no competing ontologies. The challenge with this in practice is that if there are no competing ontologies, then there is no superset of concepts and relations from which to draw into an optimal $\mathcal{O}_{c r}$ other than $O_{c r}$ itself. If we wanted to prune an ontology of its weakest parts, however, we could obtain the fittest sub-ontology $\mathcal{O}_{\gamma \phi}$, by specifying $\gamma$ concepts and $\phi$ relations so that the Depth reaches its maximum for the given $\gamma$ and $\phi$.

## 4 Comparing corpora

In addition to comparing ontologies relative to the corpora they describe, we can compare different corpora with respect to one or more ontologies. Let $T_{1}$ and $T_{2}$ indicate two distinct
corpora, such as 19th Century English novels and 20th Century scholarly medical articles. We can define the distance between the two corpora with respect to headword $h_{i}$ and its $M_{i}$ synonyms by calculating the Minkowski distance with corpora-specific parameter estimates $p_{i j}$ in the following way.

$$
\begin{equation*}
d_{T_{1}, T_{2}}\left(\mathrm{~h}_{i} \stackrel{\text { def }}{=}\left[\sum_{j=1}^{M_{i}}\left|p_{i j}^{\left(T_{1}\right)}-p_{i j}^{\left(T_{2}\right)}\right|^{r}\right]^{\frac{1}{r}} .\right. \tag{20}
\end{equation*}
$$

Or

$$
\begin{equation*}
d_{T_{1}, T_{2}}\left(\mathrm{~h}_{i}\right) \stackrel{\operatorname{def}}{=}\left[\sum_{j=1}^{M_{i}}\left|f_{i}^{\left(T_{1}\right)} p_{i j}^{\left(T_{1}\right)}-f_{i}^{\left(T_{2}\right)} p_{i j}^{\left(T_{2}\right)}\right|^{r}\right]^{\frac{1}{r}} \tag{21}
\end{equation*}
$$

In our practical implementations of this measures (we used both of the above equations in our practical experiments), we used $r=1$ (the Manhattan distance), and $r=2$ (the Eucleadean distance).

The three-way distance for three corpora, $T_{1}, T_{2}$, and $T_{3}$ is then just a sum of three pairwise distances.

$$
\begin{equation*}
d_{T_{1}, T_{2}, T_{3}}\left(\mathrm{~h}_{i}\right) \stackrel{\text { def }}{=} d_{T_{1}, T_{2}}\left(\mathrm{~h}_{i}\right)+d_{T_{1}, T_{3}}\left(\mathrm{~h}_{i}\right)+d_{T_{2}, T_{3}}\left(\mathrm{~h}_{i}\right) . \tag{22}
\end{equation*}
$$

In our three-corpus example, the most interesting headwords to visualize are those with maximum $d_{T_{1}, T_{2}, T_{3}}\left(\mathrm{~h}_{i}\right)$, which have the substitution probability estimates most unlike each other across the three corpora.

We can also define the overall distance between two corpora.

$$
\begin{equation*}
D_{T_{1}, T_{2}} \stackrel{\text { def }}{=} \sum_{i=1}^{N} d_{T_{1}, T_{2}}\left(\mathrm{~h}_{i}\right) . \tag{23}
\end{equation*}
$$

With this approach, we can compute a taxonomy or phylogeny of several corpora using a distance-matrix to construct the tree.

We can also calculate the entropy of synonyms in corpus $T$ in bits. This captures the ambiguity or linguistic richness of a corpus with respect to a thesaurus.

$$
\begin{equation*}
H_{T} \stackrel{\text { def }}{=}-\sum_{i=1}^{N} f_{i}^{(T)} \sum_{j=1}^{M_{i}} p_{i j}^{(T)} \log _{2} p_{i j}^{(T)} \tag{24}
\end{equation*}
$$

Finally, for symmetry, we can whimsically imagine the generation of a nonsense fittest corpus, which is completely consistent with a given ontology or thesaurus. That such a corpus would tend to be very redundant (or very small) highlights the limited representation most ontologies and thesauri provide of their domains, but also the collective importance of low-frequency relationships in modeling them.

## 5 Data

We used three very different corpora to illustrate our approaches.

1) Medicine: Clinical journal article abstracts from PubMed database.

Based on the clinical queries service offered by PubMed
(http://www.ncbi.nlm.nih.gov/corehtml/query/static/clinicaltable.html), we generated a modified query:

```
    ((clinical[Title/Abstract] AND trial[Title/Abstract]) OR
clinical trials[MeSH Terms] OR clinical trial[Publication Type] OR
random*[Title/Abstract] OR random allocation[MeSH Terms] OR
therapeutic use[MeSH Subheading]) OR (sensitiv*[Title/Abstract]
OR sensitivity and specificity[MeSH Terms] OR
diagnos*[Title/Abstract] OR diagnosis[MeSH:noexp] OR
diagnostic * [MeSH:noexp] OR diagnosis,differential[MeSH:noexp] OR
diagnosis[Subheading:noexp])
```

By limiting ourselves only to English abstracts in the core clinical journals for the whole period covered by PubMed, up to Feb 25, 2009, we downloaded 786,180 clinical medicinerelated abstracts.

## 2) News: Reuters News corpus

The Reuters corpus covered news stories between 08/20/1996 and 08/19/1997.
3) Literature: 19th century literature - written in English or translated to English. We compiled a subjective list of the 50 best books of the 19th century based on the information from http://www.goodreads.com/list/show/16.Best_Books_of_the_19th_Century.

We then obtain the flat text files of these books from www.gutenberg.org (see Table 1).

Table 1: Contents of the Literature corpus.

| Title | Author | English translator |
| :---: | :---: | :---: |
| Emma | Austen, Jane |  |
| Mansfield Park | Austen, Jane |  |
| Northanger Abbey | Austen, Jane |  |
| Persuasion | Austen, Jane |  |
| Pride and Prejudice | Austen, Jane |  |
| Title Sense and Sensibility | Austen, Jane |  |
| The Tenant of Wildfell Hall | Bront, Anne |  |
| Jane Eyre | Bront, Charlotte |  |
| Villette | Bront, Charlotte |  |
| Wuthering Heights | Bront, Charlotte |  |
| Alice's Adventures in Wonderland | Carroll, Lewis |  |
| Through the Looking-Glass | Carroll, Lewis |  |
| The Awakening and Selected Short Stories | Chopin, Kate |  |
| The Woman in White | Collins, Wilkie |  |
| Heart of Darkness | Conrad, Joseph |  |
| A Christmas Carol | Dickens, Charles |  |
| A Tale of Two Cities | Dickens, Charles |  |
| Bleak House | Dickens, Charles |  |
| David Copperfield | Dickens, Charles |  |
| Great Expectations | Dickens, Charles |  |
| Little Dorrit | Dickens, Charles |  |
| Our Mutual Friend | Dickens, Charles |  |
| Crime and Punishment | Dostoyevsky, Fyodor | Garnett, Constance |
| The Brothers Karamazov | Dostoyevsky, Fyodor | Garnett, Constance |


| Notes from the Underground | Dostoyevsky, Fyodor | unknown |
| :---: | :---: | :---: |
| A Study in Scarlet | Doyle, Arthur Conan, Sir |  |
| The Count of Monte Cristo | Dumas pre, Alexandre |  |
| Madame Bovary | Flaubert, Gustave | Aveling, Eleanor Marx |
| Far from the Madding Crowd | Hardy, Thomas |  |
| Tess of the d'Urbervilles | Hardy, Thomas |  |
| The Mayor of Casterbridge | Hardy, Thomas |  |
| The Scarlet Letter | Hawthorne, Nathaniel |  |
| Les Misrables | Hugo, Victor | Hapgood Isabel Florence |
| A Doll's House | Ibsen, Henrik |  |
| Moby Dick, or, the whale | Melville, Herman |  |
| Frankenstein | Shelley, Mary Wollstonecraft |  |
| Treasure Island | Stevenson, Robert Louis |  |
| Dracula | Stoker, Bram |  |
| Vanity Fair | Thackeray, William Makepeace |  |
| Anna Karenina | Tolstoy, Leo, graf | Garnett, Constance |
| War and Peace | Tolstoy, Leo, graf | Maude, Aylmer Maude, Shanks |
| A Connecticut Yankee in King Arthur's Court | Twain, Mark |  |
| Adventures of Huckleberry Finn | Twain, Mark |  |
| The Adventures of Tom Sawyer | Twain, Mark |  |
| The Prince and the Pauper | Twain, Mark |  |
| The Tragedy of Pudd' nhead Wilson | Twain, Mark |  |
| The Time Machine | Wells, H. G. (Herbert George) |  |
| The War of the Worlds | Wells, H. G. (Herbert George) |  |
| The Importance of Being Earnest | Wilde, Oscar |  |

## 6 Results

See three additional Tables with results that were not included into the main text.

Table 2: Statistics.

| Measure ${ }^{1}$ | Corpus | The synonym finder | New World Roget's A-Z thesaurus | WordNet | 21st Cen- <br> tury Syn- <br> onym And <br> Antonym  <br> Finder  | The Oxford dictionary of synonyms and antonyms | A Dictionary of Synonyms and Antonyms | Scholastic <br> Dictionary <br> of Syn- <br> onyms, <br> Antonyms <br> and <br> Homonyms |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Precision | Medicine | 0.405 | 0.335 | 0.182 | 0.543 | 0.625 | 0.576 | 0.692 |
| Precision | Novels | 0.569 | 0.424 | 0.202 | 0.718 | 0.701 | 0.833 | 0.898 |
| Precision | News | 0.610 | 0.473 | 0.261 | 0.779 | 0.807 | 0.821 | 0.876 |
| Recall | Medicine | 0.690 | 0.248 | 0.126 | 0.179 | 0.149 | 0.074 | 0.031 |
| Recall | Novels | 0.726 | 0.235 | 0.104 | 0.177 | 0.125 | 0.080 | 0.030 |
| Recall | News | 0.697 | 0.235 | 0.120 | 0.172 | 0.129 | 0.071 | 0.026 |
| Accuracy | Medicine | 0.568 | 0.594 | 0.531 | 0.683 | 0.693 | 0.680 | 0.679 |
| Accuracy | Novels | 0.641 | 0.527 | 0.430 | 0.611 | 0.595 | 0.592 | 0.576 |
| Accuracy | News | 0.635 | 0.500 | 0.405 | 0.573 | 0.561 | 0.540 | 0.524 |
| Fallout | Medicine | 0.968 | 0.314 | 0.376 | 0.079 | 0.045 | 0.027 | 0.007 |
| Fallout | Novels | 0.741 | 0.328 | 0.467 | 0.057 | 0.043 | 0.013 | 0.003 |
| Fallout | News | 0.735 | 0.331 | 0.479 | 0.049 | 0.030 | 0.015 | 0.004 |
| $F_{\beta=1}$ | Medicine | 0.510 | 0.285 | 0.149 | 0.270 | 0.240 | 0.131 | 0.059 |
| $F_{\beta=1}$ | Novels | 0.638 | 0.303 | 0.137 | 0.285 | 0.212 | 0.147 | 0.058 |
| $F_{\beta=1}$ | News | 0.651 | 0.314 | 0.165 | 0.282 | 0.222 | 0.131 | 0.051 |
| $F_{\beta=2}$ | Medicine | 0.605 | 0.262 | 0.134 | 0.207 | 0.176 | 0.090 | 0.038 |
| $F_{\beta=2}$ | Novels | 0.688 | 0.258 | 0.115 | 0.209 | 0.150 | 0.098 | 0.037 |
| $F_{\beta=2}$ | News | 0.678 | 0.261 | 0.135 | 0.204 | 0.155 | 0.087 | 0.032 |
| $F_{\beta=.5}$ | Medicine | 0.441 | 0.313 | 0.167 | 0.386 | 0.381 | 0.245 | 0.130 |
| $F_{\beta=.5}$ | Novels | 0.595 | $0.365^{3}$ | 0.170 | 0.446 | $0.364^{8}$ | 0.290 | 0.132 |
| $F_{\beta=.5}$ | News | 0.625 | $0.393^{2}$ | 0.212 | 0.457 | $0.393{ }^{1}$ | 0.264 | 0.117 |
| Breadth | Medicine | 0.521 | 0.385 | 0.260 | 0.150 | 0.284 | 0.091 | 0.060 |

[^0]| Breadth | Novels | 0.550 | 0.344 | 0.174 | 0.168 | 0.227 | 0.083 | 0.055 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Breadth | News | 0.529 | 0.369 | 0.251 | 0.158 | 0.337 | 0.098 | 0.056 |
| Breadth Loss | Medicine | 0.375 | 0.511 | 0.636 | 0.746 | 0.612 | 0.800 | 0.785 |
| Breadth Loss | Novels | 0.286 | 0.491 | 0.661 | 0.668 | 0.608 | 0.752 | 0.746 |
| Breadth Loss | News | 0.388 | 0.548 | 0.664 | 0.760 | 0.579 | 0.802 | 0.764 |
| Depth( $\cdot 10^{-6}$ ) | Medicine | 0.686 | 1.168 | 0.849 | 1.023 | 2.682 | 1.584 | 3.012 |
| Depth $\left(\cdot 10^{-6}\right)$ | Novels | 0.725 | 1.045 | 0.570 | 1.145 | 2.147 | 1.450 | 2.792 |
| Depth $\left(\cdot 10^{-6}\right)$ | News | 0.698 | 1.120 | 0.819 | 1.073 | 3.181 | 1.706 | 2.818 |
| $\operatorname{Depth}_{17, \beta=.5}\left(\cdot 10^{-4}\right)$ | Medicine | 4.314 | 4.163 | 2.399 | 1.520 | 4.651 | 1.144 | 1.033 |
| $\operatorname{Depth}_{17, \beta=.5}\left(\cdot 10^{-4}\right)$ | Novels | 4.684 | 3.521 | 1.318 | 1.799 | 3.332 | 1.001 | 0.922 |
| Depth $_{17, \beta=.5}\left(\cdot 10^{-4}\right)$ | News | 4.421 | 3.910 | 2.271 | 1.632 | 6.008 | 1.279 | 0.935 |
| Depth $_{17, \beta=.75}\left(\cdot 10^{-5}\right)$ | Medicine | 1.721 | 2.206 | 1.427 | 1.247 | 3.532 | 1.346 | 1.764 |
| Depth $_{17, \beta=.75}\left(\cdot 10^{-5}\right)$ | Novels | 1.843 | 1.918 | 0.866 | 1.435 | 2.675 | 1.205 | 1.604 |
| Depth $_{17, \beta=.75}\left(\cdot 10^{-5}\right)$ | News | 1.756 | 2.093 | 1.364 | 1.323 | 4.372 | 1.477 | 1.623 |
| Depth $_{17, \beta=1.5}\left(\cdot 10^{-8}\right)$ | Medicine | 0.109 | 0.328 | 0.301 | 0.689 | 1.546 | 2.194 | 8.783 |
| $\operatorname{Depth}_{17, \beta=1.5}\left(\cdot 10^{-8}\right)$ | Novels | 0.112 | 0.310 | 0.246 | 0.729 | 1.384 | 2.099 | 8.457 |
| Depth $_{17, \beta=1.5}\left(\cdot 10^{-8}\right)$ | News | 0.110 | 0.321 | 0.295 | 0.706 | 1.684 | 2.277 | 8.496 |
| Depth Loss ( $\cdot 10^{-5}$ ) | Medicine | 0.049 | 0.155 | 0.208 | 0.508 | 0.578 | 1.404 | 4.107 |
| $\text { Depth Loss }\left(\cdot 10^{-5}\right)$ | Novels | 0.038 | 0.149 | 0.216 | 0.455 | 0.574 | 1.312 | 3.813 |
| Depth Loss( $\cdot 10^{-5}$ ) | News | 0.051 | 0.166 | 0.217 | 0.518 | 0.548 | 1.412 | 3.935 |

Table 3: Overlaps between thesauri (headwords).

| Name $X$ | Name $Y$ | Name $Z$ | $X$ | $Y$ | $Z$ | $X \cap Y$ | $Y \cap Z$ | $X \cap Z$ | $X \cap Y \cap Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| finder | rogets | wordnet | 20,249 | 29,925 | 115,201 | 15,945 | 17,594 | 16,501 | 13,700 |
| finder | rogets | 21 century | 20,249 | 29,925 | 7,507 | 15,945 | 6,749 | 6,613 | 6,170 |
| finder | rogets | oxford | 20,249 | 29,925 | 8,487 | 15,945 | 7,498 | 7,681 | 7,103 |
| finder | rogets | synonyms | 20,249 | 29,925 | 3,771 | 15,945 | 3,540 | 3,626 | 3,457 |
| finder | rogets | scholastic | 20,249 | 29,925 | 2,147 | 15,945 | 2,044 | 2,085 | 2,018 |
| finder | wordnet | 21 century | 20,249 | 115,201 | 7,507 | 16,501 | 6,494 | 6,613 | 5,853 |
| finder | wordnet | oxford | 20,249 | 115,201 | 8,487 | 16,501 | 7,527 | 7,681 | 6,951 |
| finder | wordnet | synonyms | 20,249 | 115,201 | 3,771 | 16,501 | 3,429 | 3,626 | 3,335 |
| finder | wordnet | scholastic | 20,249 | 115,201 | 2,147 | 16,501 | 1,966 | 2,085 | 1,929 |
| finder | 21 century | oxford | 20,249 | 7,507 | 8,487 | 6,613 | 4,101 | 7,681 | 3,914 |


| finder | 21 century | synonyms | 20,249 | 7,507 | 3,771 | 6,613 | 2,231 | 3,626 | 2,205 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| finder | 21 century | scholastic | 20,249 | 7,507 | 2,147 | 6,613 | 1,359 | 2,085 | 1,343 |
| finder | oxford | synonyms | 20,249 | 8,487 | 3,771 | 7,681 | 2,470 | 3,626 |  |
| finder | oxford | scholastic | 20,249 | 8,487 | 2,147 | 7,681 | 1,652 | 2,085 | 1,641 |
| finder | synonyms | scholastic | 20,249 | 3,771 | 2,147 | 3,626 | 1,259 | 2,085 | 1,249 |
| rogets | wordnet | 21 century | 29,925 | 115,201 | 7,507 | 17,594 | 6,494 | 6,749 | 5,930 |
| rogets | wordnet | oxford | 29,925 | 115,201 | 8,487 | 17,594 | 7,527 | 7,498 | 6,792 |
| rogets | wordnet | synonyms | 29,925 | 115,201 | 3,771 | 17,594 | 3,429 | 3,540 | 3,261 |
| rogets | wordnet | scholastic | 29,925 | 115,201 | 2,147 | 17,594 | 1,966 | 2,044 | 1,892 |
| rogets | 21 century | oxford | 29,925 | 7,507 | 8,487 | 6,749 | 4,101 | 7,498 | 3,846 |
| rogets | 21 century | synonyms | 29,925 | 7,507 | 3,771 | 6,749 | 2,231 | 3,540 | 2,180 |
| rogets | 21 century | scholastic | 29,925 | 7,507 | 2,147 | 6,749 | 1,359 | 2,044 | 1,334 |
| rogets | oxford | synonyms | 29,925 | 8,487 | 3,771 | 7,498 | 2,470 | 3,540 | 2,406 |
| rogets | oxford | scholastic | 29,925 | 8,487 | 2,147 | 7,498 | 1,652 | 2,044 | 1,624 |
| rogets | synonyms | scholastic | 29,925 | 3,771 | 2,147 | 3,540 | 1,259 | 2,044 | 1,244 |
| wordnet | 21 century | oxford | 115,201 | 7,507 | 8,487 | 6,494 | 4,101 | 7,527 | 3,679 |
| wordnet | 21 century | synonyms | 115,201 | 7,507 | 3,771 | 6,494 | 2,231 | 3,429 | 2,063 |
| wordnet | 21 century | scholastic | 115,201 | 7,507 | 2,147 | 6,494 | 1,359 | 1,966 | 1,251 |
| wordnet | oxford | synonyms | 11,201 | 8,487 | 3,771 | 7,527 | 2,470 | 3,429 | 2,307 |
| wordnet | oxford | scholastic | 115,201 | 8,487 | 2,147 | 7,527 | 1,652 | 1,966 | 1,543 |
| wordnet | synonyms | scholastic | 115,201 | 3,771 | 2,147 | 3,429 | 1,259 | 1,966 | 1,174 |
| 21 century | oxford | synonyms | 7,507 | 8,487 | 3,771 | 4,101 | 2,470 | 2,231 | 1,558 |
| 21 century | oxford | scholastic | 7,507 | 8,487 | 2,147 | 4,101 | 1,652 | 1,359 | 1,080 |
| 21 century | synonyms | scholastic | 7,507 | 3,771 | 2,147 | 2,231 | 1,259 | 1,359 | 885 |
| oxford | synonyms | scholastic | 8,487 | 3,771 | 2,147 | 2,470 | 1,259 | 1,652 | 1,053 |

Table 4: Overlaps between thesauri (synonym pairs).

| Name $X$ | Name $Y$ | Name $Z$ | $X$ | $Y$ | $Z$ | $X \cap Y$ | $Y \cap Z$ | $X \cap Z$ | $X \cap Y \cap Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| finder | rogets | wordnet | 758,611 | 329,669 | 306,472 | 97,204 | 20,804 | 39,094 | 14,591 |
| finder | rogets | 21 century | 758,611 | 329,669 | 146,806 | 97,204 | 46,323 | 72,833 | 28,093 |
| finder | rogets | oxford | 758,611 | 329,669 | 105,902 | 97,204 | 30,914 | 56,054 |  |
| finder | rogets | synonyms | 758,611 | 329,669 | 57,366 | 97,204 | 21,821 | 32,390 | 15,985 |
| finder | rogets | scholastic | 758,611 | 329,669 | 19,759 | 97,204 | 7,650 | 13,031 | 6,422 |
| finder | wordnet | 21 century | 758,611 | 306,472 | 146,806 | 39,094 | 13,511 | 72,833 | 9,942 |
| finder | wordnet | oxford | 758,611 | 306,472 | 105,902 | 39,094 | 13,714 | 56,054 | 10,292 |
| finder | wordnet | synonyms | 758,611 | 306,472 | 57,366 | 39,094 | 6,000 | 32,390 | 5,167 |
| finder | wordnet | scholastic | 758,611 | 306,472 | 19,759 | 39,094 | 2,959 | 13,031 | 2,617 |


|  | finder | 21 century | oxford | 758,611 | 146,806 | 105,902 | 72,833 | 24,624 | 56,024 | 18,300 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | finder | 21 century | synonyms | 758,611 | 146,806 | 57,366 | 72,833 | 15,787 | 32,390 | 12,390 |
|  | finder | 21 century | scholastic | 758,611 | 146,806 | 19,759 | 72,833 | 6,804 | 13,031 | 5,622 |
|  | finder | oxford | synonyms | 758,611 | 105,902 | 57,366 | 56,024 | 10,617 | 32,390 | 9,217 |
|  | finder | oxford | scholastic | 758,611 | 105,902 | 19,759 | 56,024 | 5,347 | 13,031 | 4,747 |
|  | finder | synonyms | scholastic | 758,611 | 57,366 | 19,759 | 32,390 | 7,521 | 13,031 | 6,091 |
|  | rogets | wordnet | 21 century | 329,669 | 306,472 | 146,806 | 20,804 | 13,511 | 46,323 | 6,003 |
|  | rogets | wordnet | oxford | 329,669 | 306,472 | 105,902 | 20,804 | 13,714 | 30,914 | 6,699 |
|  | rogets | wordnet | synonyms | 329,669 | 306,472 | 57,366 | 20,804 | 6,000 | 21,821 | 3,499 |
|  | rogets | wordnet | scholastic | 329,669 | 306,472 | 19,759 | 20,804 | 2,959 | 7,650 | 1,749 |
|  | rogets | 21 century | oxford | 329,669 | 146,806 | 105,902 | 46,323 | 24,624 | 30,914 | 11,178 |
|  | rogets | 21 century | synonyms | 329,669 | 146,806 | 57,366 | 46,323 | 15,787 | 21,821 | 8,801 |
|  | rogets | 21 century | scholastic | 329,669 | 146,806 | 19,759 | 46,323 | 6,804 | 7,650 | 3,577 |
|  | rogets | oxford | synonyms | 329,669 | 105,902 | 57,366 | 30,914 | 10,617 | 21,821 | 6,559 |
|  | rogets | oxford | scholastic | 329,669 | 105,902 | 19,759 | 30,914 | 5,347 | 7,650 | 3,297 |
|  | rogets | synonyms | scholastic | 329,669 | 57,366 | 19,759 | 21,821 | 7,521 | 7,650 | 4,292 |
|  | wordnet | 21 century | oxford | 306,472 | 146,806 | 105,902 | 13,511 | 24,624 | 13,714 | 4,718 |
|  | wordnet | 21 century | synonyms | 306,472 | 146,806 | 57,366 | 13,511 | 15,787 | 6,000 | 2,667 |
|  | wordnet | 21 century | scholastic | 306,472 | 146,806 | 19,759 | 13,511 | 6,804 | 2,959 | 1,462 |
| $\stackrel{\rightharpoonup}{\square}$ | wordnet | oxford | synonyms | 306,472 | 105,902 | 57,366 | 13,714 | 10,617 | 6,000 | 2,664 |
|  | wordnet | oxford | scholastic | 306,472 | 105,902 | 19,759 | 13,714 | 5,347 | 2,959 | 1,563 |
|  | wordnet | synonyms | scholastic | 306,472 | 57,366 | 19,759 | 6,000 | 7,521 | 2,959 | 1,543 |
|  | 21 century | oxford | synonyms | 146,806 | 105,902 | 57,366 | 24,624 | 10,617 | 15,787 | 4,748 |
|  | 21 century | oxford | scholastic | 146,806 | 105,902 | 19,759 | 24,624 | 5,347 | 6,804 | 2,570 |
|  | 21 century | synonyms | scholastic | 146,806 | 57,366 | 19,759 | 15,787 | 7,521 | 6,804 | 3,432 |
|  | oxford | synonyms | scholastic | 105,902 | 57,366 | 19,759 | 10,617 | 7,521 | 5,347 | 2,963 |


[^0]:    ${ }^{1}$ Changes in ranking of a measure across three corpora are highlighted in red. Font size reflects the ranking of results, the best results shown with the largest font, the worst with the smallest.

