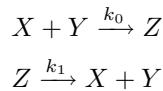


**Table S1: Parameter ranges and non-dimensionalization**

| Parameter | Description  | Biological range         |                 | Scaling parameter      |           | Non-dimensional |
|-----------|--|--------------------------|-----------------|------------------------|-----------|-----------------|
| $b_{syn}$ | basal synthesis rate                               | $10^{-12}$               | $M s^{-1}$      | $\alpha\beta$          | $10^{10}$ | $M^{-1} s$      |
| $k_{deg}$ | basal degradation rate constant                    | $10^{-5}$                | $s^{-1}$        | $\alpha$               | $10^3$    | $s$             |
| $K_P$     | basal activation Michaelis constant                | $10^{-10} \dots 10^{-6}$ | $M$             | $\beta$                | $10^7$    | $M^{-1}$        |
| $k_P$     | basal activation catalytic rate constant           | $10^{-2} \dots 10^2$     | $s^{-1}$        | $\alpha$               | $10^3$    | $s$             |
| $K_Q$     | basal inactivation Michaelis constant              | $10^{-10} \dots 10^{-6}$ | $M$             | $\beta$                | $10^7$    | $M^{-1}$        |
| $k_Q$     | basal inactivation catalytic rate constant         | $10^{-2} \dots 10^2$     | $s^{-1}$        | $\alpha$               | $10^3$    | $s$             |
| $k_0$     | complex association rate constant                  | $10^6 \dots 10^7$        | $M^{-1} s^{-1}$ | $\frac{\alpha}{\beta}$ | $10^{-4}$ | $Ms$            |
| $k_1$     | complex dissociation rate constant                 | $10^{-3} \dots 10^1$     | $s^{-1}$        | $\alpha$               | $10^3$    | $s$             |
| $k_2$     | catalytic rate constant                            | $10^{-2} \dots 10^2$     | $s^{-1}$        | $\alpha$               | $10^3$    | $s$             |
| $v$       | maximal transcription rate                         | $10^{-13} \dots 10^{-9}$ | $Ms^{-1}$       | $\alpha\beta$          | $10^{10}$ | $M^{-1} s$      |
| $K_{syn}$ | conc. at which transcription rate is $\frac{v}{2}$ | $10^{-8} \dots 10^{-6}$  | $M$             | $\beta$                | $10^7$    | $M^{-1}$        |
| $P$       | basal activator concentration                      | $10^{-9}$                | $M$             | $\beta$                | $10^7$    | $M^{-1}$        |
| $Q$       | basal inactivator concentration                    | $10^{-8}$                | $M$             | $\beta$                | $10^7$    | $M^{-1}$        |
|           |  |                          |                 |                        |           | 0.1             |

As an example, a simple association and dissociation reaction with species X, Y, and complex Z would be non-dimensionalized as follows.



$$Y_t = Y + Z$$

$$\hat{Y} = \frac{Y}{Y_t}$$

$$\hat{Z} = \frac{Z}{Y_t}$$

$$\begin{aligned} \frac{dZ}{dt} &= k_0 XY - k_1 Z \\ \frac{d\hat{Z}}{dt} &= k_0 X \hat{Y} - k_1 \hat{Z} \\ \alpha \frac{d\hat{Z}}{dt} &= \alpha k_0 X \hat{Y} - \alpha k_1 \hat{Z} \\ \alpha \frac{d\hat{Z}}{dt} &= \frac{\alpha k_0}{\beta} (\beta X) \hat{Y} - \alpha k_1 \hat{Z} \end{aligned}$$

Here,  $k_0$  is multiplied by the scaling parameter  $\frac{\alpha}{\beta}$  to obtain a non-dimensional  $k_0$  and, similarly,  $k_1$  is multiplied by  $\alpha$  to obtain a non-dimensional  $k_1$  (where  $\alpha = 10^3 s$  and  $\beta = 10^7 M^{-1}$  are constants). Repeating this non-dimensionalization procedure on the entire set of model equations yields the list of scaling parameters and non-dimensional parameter ranges in the table above.