

Text S1: Mass-action killing and the interpretation of the rate constant k

In a region of tissue containing N cells, suppose a proportion f_C are CTL and a proportion f_I are infected cells, we assume f_I is small and sufficiently small that susceptible cells are abundant, and the infected cell population is growing at rate r . These proportions are the densities referred to in the text. Then a mass-action model of killing of these infected cells by CTL is

$$\frac{d(f_I N)}{dt} = r f_I N - K(f_C N)(f_I N). \quad (1)$$

This assumes that CTL and infected cells are well mixed and moving randomly. The interpretation of the last term is as follows; $K(f_C N)$ is the probability per unit time (the rate parameter defining the underlying Poisson process) of any particular cell, infected or not, being encountered by any CTL; K is the probability per unit time of a given CTL encountering this cell; and KN is the probability per unit time of a given CTL encountering any cell at all; this is the rate of surveillance by a single CTL. The expected number of cells encountered in unit time by one CTL is then $1/(KN)$.

Equation (1) is then

$$\frac{df_I}{dt} = r f_I - k f_C f_I. \quad (2)$$

where $k = KN$. This parameter k is the CTL surveillance rate (total number of cells surveyed per unit time per CTL) estimated in [1] and [2].

When C and I are measured as cells per unit volume, the interpretation of the constant of proportionality k in the loss rate kCI is different; k now contains dimensions of volume and so kI is the rate that a single CTL encounters infected cells present at a density I per unit volume. If the total density of surveyable cells in a tissue is S , kS can be interpreted as the rate of surveillance.

References

1. Regoes RR, Barber DL, Ahmed R, Antia R (2007) Estimation of the rate of killing by cytotoxic T lymphocytes in vivo. Proc Natl Acad Sci U S A 104: 1599-603.
2. Yates A, Graw F, Barber DL, Ahmed R, Regoes RR, et al. (2007) Revisiting estimates of CTL killing rates in vivo. PLoS One 2: e1301.