Text S3: Derivation of the Lotka-Euler equation

The Lotka-Euler equation (Equation 16) can be derived as follows. The total number of infected cells in age class a at time t is the cells that were infected a time a ago and survived to age a;

$$I(a,t) = \ell(a)I(0, t-a) = \epsilon\ell(a) \int_{s=0}^{\infty} I(s, t-a)m(s)ds$$

We are assuming steady state exponential growth of infected cells in all age-classes, $I(a,t) = e^{rt}I(a,0)$, so

$$
\begin{aligned}
I(a,t) &= \epsilon\ell(a) \int_{s=0}^{\infty} I(s,t)e^{-ra}m(s)ds \\
&= \epsilon\ell(a)e^{-ra} \int_{s=0}^{\infty} I(s,t)m(s)ds
\end{aligned}
$$

The total rate of virus production at time t is then

$$\int_a I(a,t)m(a)da = \epsilon \int_a m(a)\ell(a)e^{-ra}da \int_s I(s,t)m(s)ds$$

which gives Equation 16 in the main text.