**S.3 Polygonal Path Approximation**

 

1. (b)

Figure S2: A zoom-in view of the traced result. (a) Red lines show the tracing result; (b) The *ϵ*-approximation of the tracing result, where *ϵ* =$\sqrt{3}$.

Given a polygonal path *S* =*< v*0*, . . . , vm >* and an error bound *ϵ*, we look for a polygonal path, $\tilde{S}$,that is an *ϵ*-approximation of *S*. $\tilde{S}$=*< u*0*, . . . , um >* optimally *ϵ*-approximates *S* if $\tilde{S}$meets the following criteria.

1. Vertex set of $\tilde{S}$is a subset of *S*.
2. Let *ui* = *vj* and *ui*+1 = *vk, i* = 1*, . . . ,m -*1, the distance between any vertex on the polygonal path *< vj . . . , vk >* and the line segment *< ui, ui*+1 *>* is less than *ϵ*.
3. The number of the vertices on $\tilde{S}$is the smallest possible.

This problem can be solved using the dynamic programming technique. We define the number of edges on $\tilde{S}(i, j)$ to be its cost. The lowest cost among all the *ϵ*-approximations for *S* is the optimal cost, denoted *c*(*i, j*). For the boundary condition that *i* = *j*, we let *c*(*i, j*) = 0. If *j > i*, there are two cases for establishing the optimal *ϵ*-approximation path.

* Case 1: $\tilde{S}(i, j)$ is the line segment (*vi*, *vj*)

This case occurs when all the distances between vertices *vk*, *i* ≤ *k* ≤ *j*, to
(*vi*, *vj*) are less than *ϵ*. (*vi*, *vj*) *ϵ*-approximates *< vi, vi+*1*, . . . , vj >*, and thus
*c*(*i*, *j*) = 1

* Case 2: $\tilde{S}(i, j)$ consists of two or more line segments.

I In this case, $\tilde{S}(i, j)$ can be divided into two sub-paths, $\tilde{S}(i, k)$ and $\tilde{S}(k, j)$, where *vk* is a vertex on *< vi, . . . , vj >*. Note that both $\tilde{S}(i, k)$ and $\tilde{S}(k, j)$ *ϵ*-approximate polygonal paths *< vi, . . . , vk >* and *< vk, . . . , vj >*. The cost of optimal *ϵ*-approximation *c*(*i, j*) is $\min\_{i<k<j}(c\left(i,k\right)+c\left(k,j\right))$.

Based on the above discussion, the optimal cost can be written in the recurrence

$$c\left(i, j\right)=\left\{\begin{array}{c}0\\1\\\min\_{i<k<j}\left(c\left(i,k\right)+c\left(k,j\right)\right)\end{array}\right.\begin{matrix} if i=j\\ if \left(v\_{i},v\_{j}\right) ϵ-approximates<v\_{i},…,v\_{j}> (1)\\ otherwise\end{matrix}$$

Solving the dynamic programming, we compute the optimal *ϵ*-approximation of a polygonal path.