Dynamical systems: dealing with unknown delays

This note complements the section "Dealing with delays" of the main manuscript. The idea we develop here is construct a *corrected* evolution function, using a first-order Taylor expansion around zero-delays (see, e.g., [32]).

Let us assume that the system obeys the following ODE (in continuous time):

 (A1)

Now let us Taylor-expand the evolution function around :

 (A2)

Replacing Equation A2 into Equation A1 now yields:

 (A3)

which can then be used to derive a "delay-corrected" evolution function in discrete time. For example, an Euler discretization scheme now yields:

 (A4)

where  is the time discretization step and the state noise  lumps together  error terms and any other perturbations. Note that in this formulation, delays can now be estimated as part of the evolution parameters ().

The VBA toolbox does not (yet) automatically perform such analytical approximation of delayed dynamics. This means that one has to implement Equation A4 directly in the system's evolution function.