VB approximate inference: dealing with categorical observations

First, let us consider a simple generative model (with no hidden states) dealing with binary data variables. Without loss of generality, the likelihood function can be written as the following Bernouilli conditional distribution:

 (A5)

where  is a binary data point (there are  of these), whose first-order moment  is determined by model parameters, i.e.: . By analogy with Equations 1-3 in the main text, we will refer to the function  as the observation mapping.

Given priors  on model parameters, any arbitrary probability density function  induces a free-energy bound on the log-model evidence:

 (A6)

where we have used the Laplace approximation, yielding a Gaussian fixed-form approximate posterior density . Under Normal priors (), the gradient and Hessian of the log- joint density  then write:

 (A7)

where the second-order derivatives of the observation mapping have been neglected. This is actually necessary in the general case, because it can potentially induce non positive-definite covariance matrices. This yields an analytical form for the second-order moment of the approximate (Laplace) posterior density:

 (A8)

where the divide and square operators within the brackets of the right-hand-side of Equation A8 improperly denote the element-by-element operations on the appropriate vectors.

From here, the extension to a nonlinear stochastic state-space model is almost trivial. One simply needs to replace the log-density log- joint density  by its expectation over the appropriate model variable partition. This is what the VBA toolbox automatically does when *options.binomial* is set to 1.

Note that when dealing with categorical (binomial) observations, the precision  of model residuals becomes irrelevant. However, one can still eyeball estimated model residuals  and their autocorrelation function from the "diagnostics" tab (cf. "VBA\_ReDisplay.m"). Although these residuals are now bounded between -1 and 1, their interpretation is similar to the continuous (gaussian) case.